B.Sc-part-II (Sub). Diff. Calculus. 11-07-2020

Partial ~~Differentiation~~ derivatives of two independent variables

Q1. Prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$ when $u=\sin^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y}{x}\right)$

Soln. We have, $u=\sin^{-1}\left[\frac{x}{y}\right]+\tan^{-1}\left(\frac{y}{x}\right)$

Diff. both sides with respect to x, we get

$$\frac{\partial u}{\partial x}=\frac{1}{\sqrt{1-\left(\frac{x}{y}\right)^2}}\cdot\frac{1}{y}+\frac{1}{1+\left(\frac{y}{x}\right)^2}\left(-\frac{y}{x^2}\right)$$

$$=\frac{1}{\sqrt{y^2-x^2}}\times\frac{\cancel{y}}{\cancel{y}}-\frac{1}{x^2+y^2}\times\frac{\cancel{x^2}y}{\cancel{x^2}}$$

$$=\frac{1}{\sqrt{y^2-x^2}}-\frac{y}{x^2+y^2}$$

Again diff. with respect to y in both sides, we get

$$\frac{\partial u}{\partial y}=\frac{1}{\sqrt{1-\left(\frac{x}{y}\right)^2}}\left(-\frac{x}{y^2}\right)+\frac{1}{1+\left(\frac{y}{x}\right)^2}\times\frac{1}{x}$$

$$=-\frac{y}{\sqrt{y^2-x^2}}\times\frac{x}{y^2}+\frac{x^{\cancel{2}}}{x^2+y^2}\times\frac{1}{\cancel{x}}$$

$$=\frac{-x}{y\sqrt{y^2-x^2}}+\frac{x}{x^2+y^2}$$

$$\therefore x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{x}{\sqrt{y^2-x^2}}-\cancel{\frac{xy}{x^2+y^2}}-\frac{xy}{y\sqrt{y^2-x^2}}+\cancel{\frac{xy}{x^2+y^2}}$$

$$=\frac{x}{\sqrt{y^2-x^2}}-\frac{x}{\sqrt{y^2-x^2}}$$

$$\Rightarrow x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$$

Proved.

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Partial derivative of two independent variables.

Q. 2. If $u = \log \frac{x^2+y^2}{x+y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

Soln: We have, $u = \log\frac{(x^2+y^2)}{(x+y)} = \log(x^2+y^2) - \log(x+y)$

~~$\therefore \frac{\partial u}{\partial x} = \frac{1}{x^2+y^2} \cdot 2x$~~

Diff. both sides with respect to x, we get

$$\frac{\partial u}{\partial x} = \frac{1}{x^2+y^2} \times 2x - \frac{1}{x+y} \times 1$$

$$\therefore x\frac{\partial u}{\partial x} = \frac{2x^2}{x^2+y^2} - \frac{x}{x+y} \quad \text{——— (I)}$$

Also, $u = \log(x^2+y^2) - \log(x+y)$

Diff. both sides with respect to y, we get

$$\frac{\partial u}{\partial y} = \frac{1}{x^2+y^2} \times 2y - \frac{1}{x+y} \times 1$$

$$\therefore y\frac{\partial u}{\partial y} = \frac{2y^2}{x^2+y^2} - \frac{y}{x+y} \quad \text{——— (2)}$$

Adding (1) and (2), we get

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\left[\frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2}\right] - \left[\frac{x}{x+y} + \frac{y}{x+y}\right]$$

$$= 2\,\frac{\cancel{x^2+y^2}}{\cancel{x^2+y^2}} - \frac{\cancel{x+y}}{\cancel{x+y}} = 2 - 1 = 1$$

Hence, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$ proved

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